Comparison of multi-scale methods for modeling perforated plate in computational structural mechanics

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Introduction

Riveted assembly modeling

- **Physical phenomena**
  - → Stress, strain concentrated
- **Difficulties**
  - → Incompatibility (assembly size vs structure)
- **Multiscale methods**
  - → Super-element: Hybrid-Trefftz displacement (HT-D) element
  - → Homogenization: Integrated Transformation Fields Analysis (TFA) and combined TFA – Mori-Tanaka (MT)

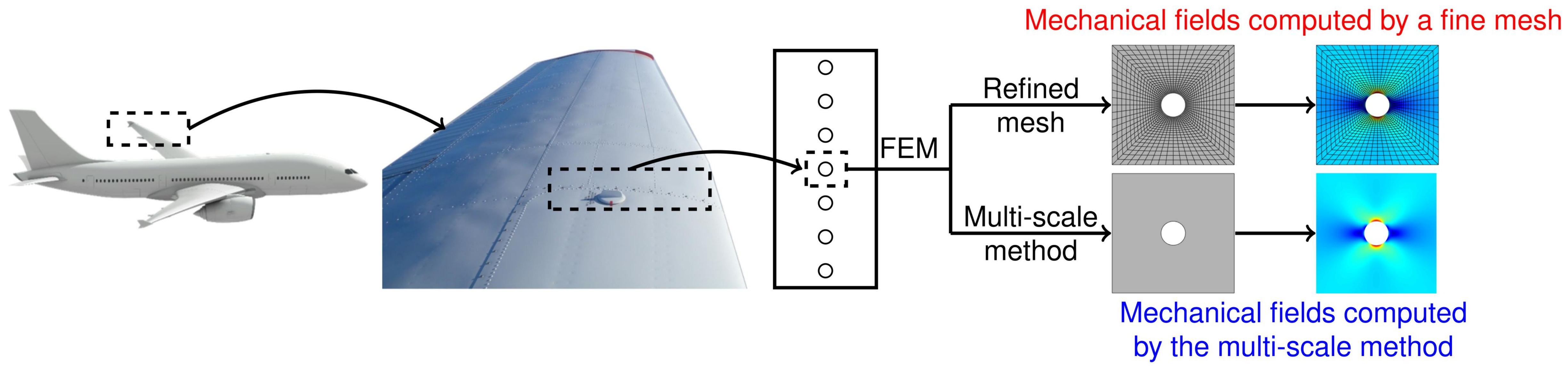


Figure 1: *Modeling of riveted assembly areas of an aeronautical structure (by a fine mesh or a multi-scale method)*

Hybrid-Trefftz displacement element

HT-D principle

- **Boundary field**: Compatibility with the neighboring FE
- **Interior field**: Kolosov-Muskhelishvili (KM) analytical solution for linear perforated membrane

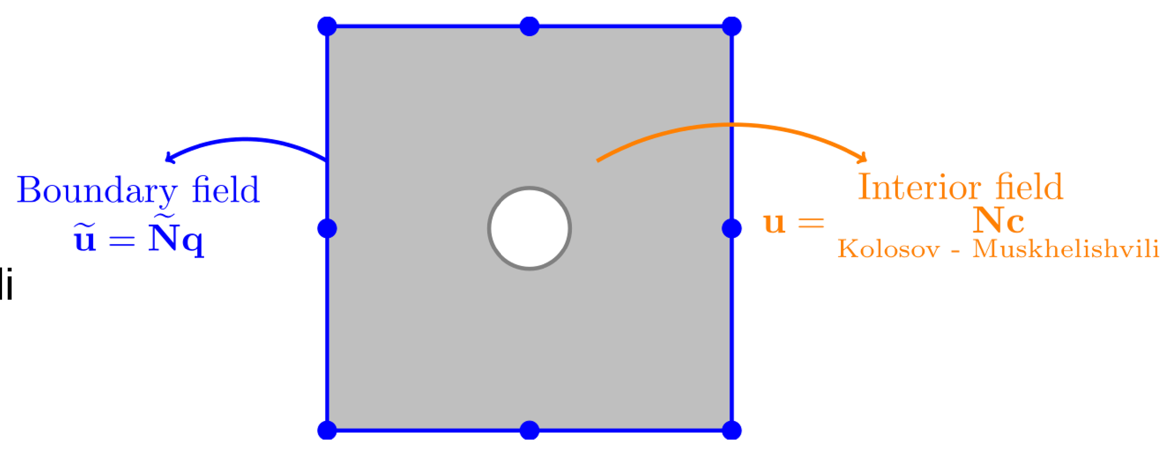


Figure 2: *Displacement fields in the HT-D method*

Practical use

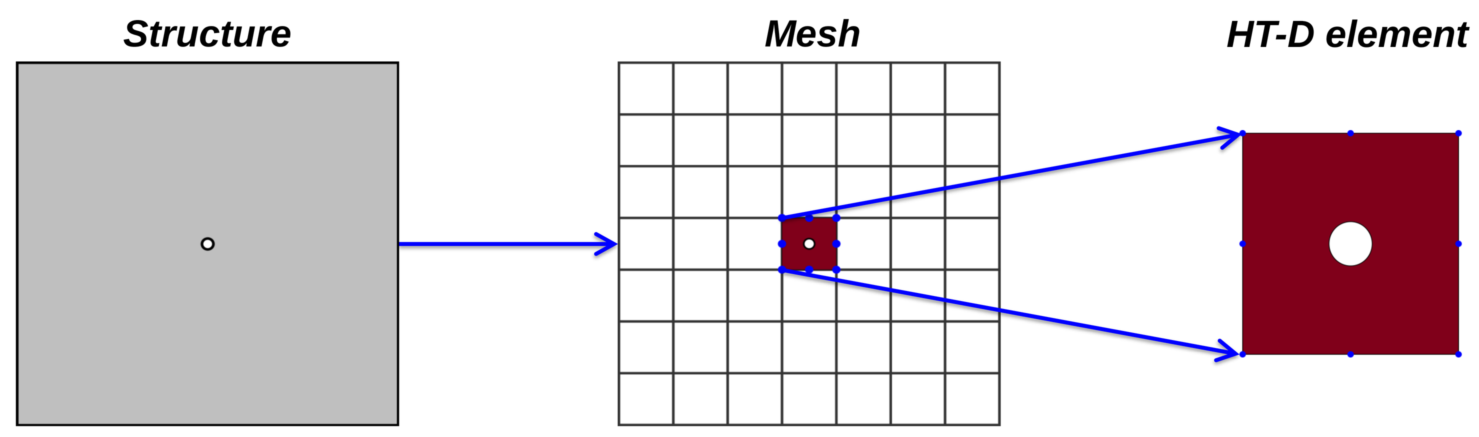


Figure 3: *Using the HT-D element in structural computations*

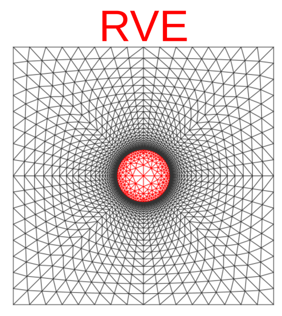
Homogenization method (TFA and combined TFA – MT)

TFA integrated homogenization method

$\boldsymbol{E}$ → **(1) Localization step**

$$\boldsymbol{\varepsilon}_r = \boldsymbol{A}_r : \boldsymbol{E} + \sum_{s=1}^{N} \boldsymbol{D}_{rs} : \boldsymbol{\varepsilon}_s^{in}$$

RVE



(3) Homogenization step

$$\boldsymbol{\Sigma} = \sum_{r=1}^{N} c_r \boldsymbol{\sigma}_r$$

Refined mesh (N sub volumes) ⇒ a priori expensive

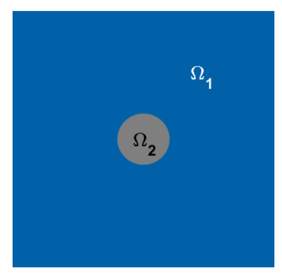
$\boldsymbol{\varepsilon}_r$ → **(2) Integration step** $\boldsymbol{\sigma}_r = \boldsymbol{f}(\boldsymbol{\varepsilon}_r)$ → $\boldsymbol{\sigma}_r$

MT integrated homogenization method

$\boldsymbol{E}$ → **(1) Localization step**

$$\boldsymbol{\varepsilon}_{1,2} = \boldsymbol{A}_{1,2} : \boldsymbol{E} + \sum_{s=1}^{2} \boldsymbol{D}_{1s,2s} : \boldsymbol{\varepsilon}_s^{in}$$

RVE



(3) Homogenization step

$$\boldsymbol{\Sigma} = c_1 \boldsymbol{\sigma}_1 + c_2 \boldsymbol{\sigma}_2$$

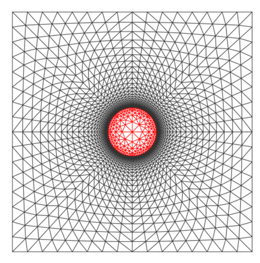
2 sub volumes ⇒ no localization

$\boldsymbol{\varepsilon}_{1,2}$ → **(2) Integration step** $\boldsymbol{\sigma}_{1,2} = \boldsymbol{f}(\boldsymbol{\varepsilon}_{1,2})$ → $\boldsymbol{\sigma}_{1,2}$

Combined integrated TFA – MT

$$\boldsymbol{\varepsilon}_r = \boldsymbol{A}_r : \boldsymbol{E} + \sum_{s=1}^{N} \boldsymbol{D}_{rs} : \boldsymbol{\varepsilon}_s^{in}$$

TFA numerical localization



$$\boldsymbol{\sigma}_r = \boldsymbol{B}_r : \boldsymbol{\Sigma} - \sum_{s=1}^{N} \boldsymbol{F}_{rs} : \boldsymbol{L}_s : \boldsymbol{\varepsilon}_s^{in}$$

TFA numerical localization

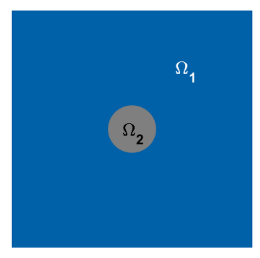
(4) Post-processing

$\boldsymbol{E}$ — Homogenization — $\boldsymbol{\Sigma}$

(1) Localization step

$$\boldsymbol{\varepsilon}_{1,2} = \boldsymbol{A}_{1,2} : \boldsymbol{E} + \sum_{s=1}^{2} \boldsymbol{D}_{1s,2s} : \boldsymbol{\varepsilon}_s^{in}$$

Mori-Tanaka



(3) Homogenization step

$$\boldsymbol{\Sigma} = c_1 \boldsymbol{\sigma}_1 + c_2 \boldsymbol{\sigma}_2$$

Mori-Tanaka

$\boldsymbol{\varepsilon}_{1,2}$ → **(2) Integration step** $\boldsymbol{\sigma}_{1,2} = \boldsymbol{f}(\boldsymbol{\varepsilon}_{1,2})$ → $\boldsymbol{\sigma}_{1,2}$

Practical use

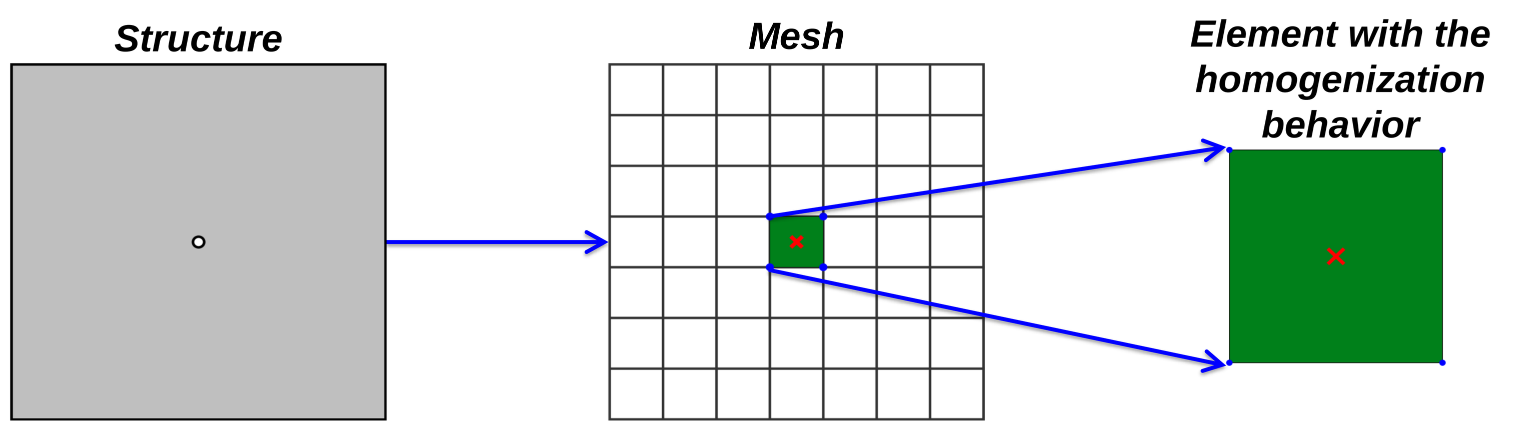


Figure 4: *Homogenization method in structural computations*

Numerical results

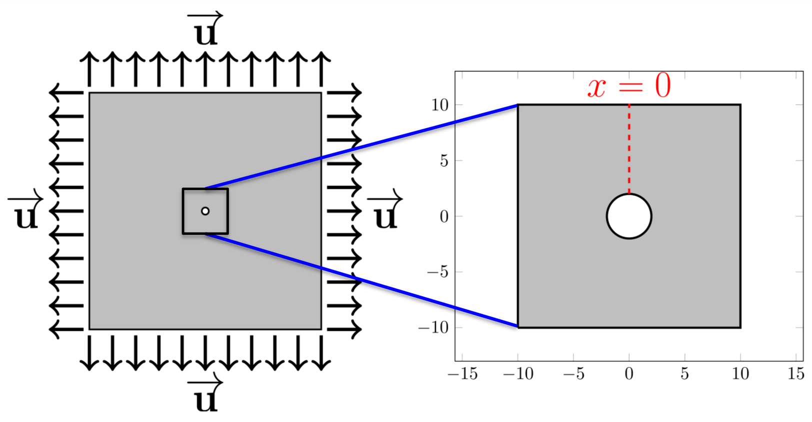


Figure 5: *Test case and post-processing subdomain*

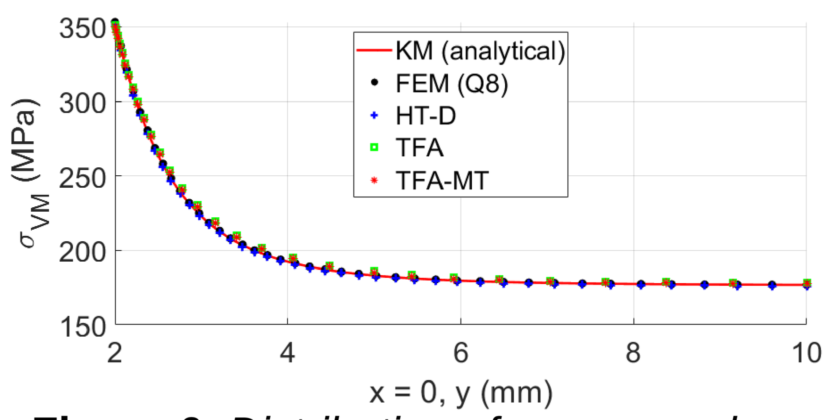


Figure 6: *Distribution of $\sigma_{von\ Mises}$ along the $\vec{y}$ axis*

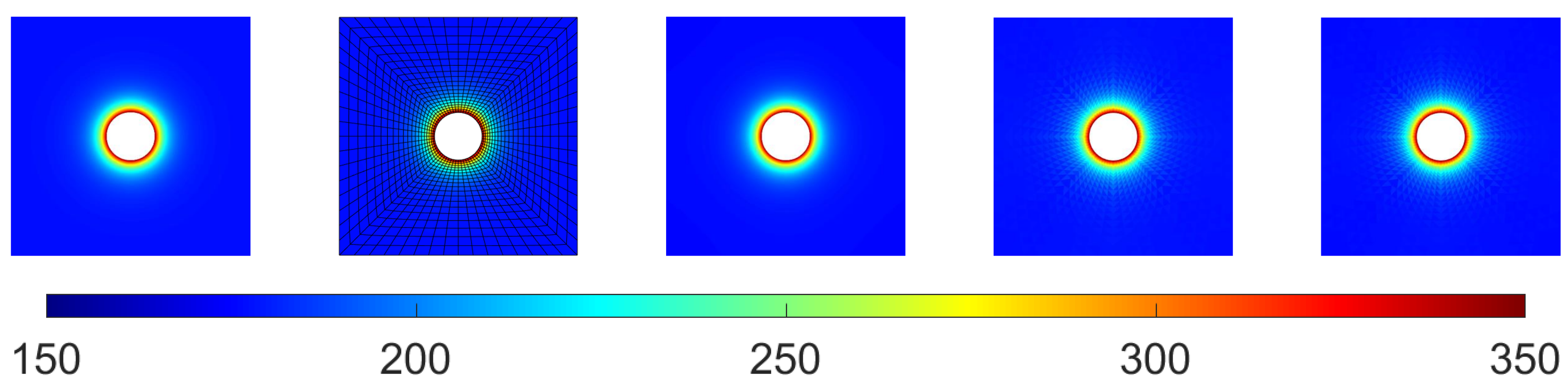


Figure 7: *Distribution of the von Mises stress over the post-processing subdomain. From left to right: KM (analytical), FEM (Q8), HT-D, TFA and TFA – MT*

Method	$\max(\sigma_{von\ Mises})$ (MPa)	Error %	Computational time (s)
KM (analytical)	353.0	-	-
FEM (Q8)	353.2	0.06	5.5
HT-D	351.3	0.5	0.08
TFA	351.3	0.5	980
TFA – MT	350.0	0.8	0.07

Conclusion and prospects

- All methods were able to correctly localize the fields in the linear structural multiscale model.
- The HT-D super-element was very accurate in the test case, but it is limited to linear computations.
- The TFA method was too costly to be used in modeling the perforated structure.
- The combined TFA – MT method exhibited similar efficiency to the HT-D method.
- To address materially nonlinear problems, only the combined TFA – MT method seems promising in terms of cost-effectiveness.

References

[1] N. Leconte, B. Langrand, and E. Markiewicz, "On some features of a plate hybrid-Trefftz displacement element containing a hole," *Finite Elements in Analysis and Design*, vol. 46, no. 10, pp. 819–828, Oct. 2010.

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