Super-élément fini non-linéaire pour la modélisation des assemblages dans les calculs de structures

Non-linear super finite element for the modeling of assemblies in structural computations

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Context and objectives

Riveted assembly areas play a crucial role in the resistance of mechanical structures subjected to shocks and impacts, particularly in the aeronautical sector. A detailed numerical simulation analysis of the generated physical phenomena necessitates precise modeling of both the entire structure and the assembly zone. Despite advancements in computing power, simulations of this nature remain complex due to the need for highly refined modeling in assembly areas, which generates a stable time step that is often incompatible with the numerical simulation of an aeronautical structure. To address this challenge, an approach based on modeling assembly zones using super-elements was therefore considered. These elements rely on the hybrid-Trefftz displacement (HT-D) principle and use the perforated membrane solution of Kolosov-Muskhelishvili [1]. Such elements have demonstrated considerable effectiveness in the linear elastic domain. Consequently, this research aims to develop an analogous formulation for materially and geometrically non-linear problems. Different multi-scale methods (such as homogenization), model reduction (Proper Orthogonal Decomposition) or machine learning (neural networks) are therefore considered to meet this need. Firstly, the objective is to evaluate the effectiveness of these methods (precision of field localization, computation times) in a linear framework. Next, the most promising approaches will be selected for non-linear problems.

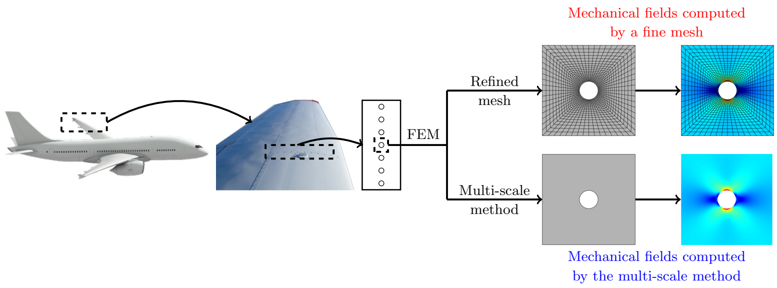

Figure 1: Modeling of riveted assembly areas of an aeronautical structure (by a fine mesh or a multi-scale method)

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Problematic

The formulation of the super-element is based on the hybrid-Trefftz principle. Within this framework, the displacement field is split into two independent components: a displacement field within the element $\mathbf{u}$, which satisfies the equilibrium equation a priori, and an inter-element displacement field $\tilde{\mathbf{u}}$, which ensures the continuity condition of the displacements between the elements (fig. 2). It is chosen to be compatible with conventional elements.

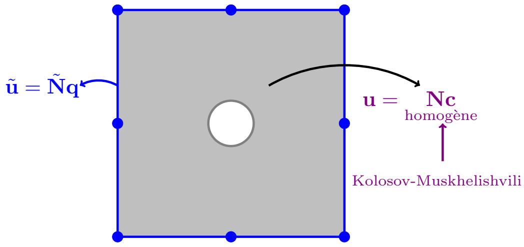

Figure 2: Displacement fields in the hybrid-Trefftz method

Expressing interior fields as a linear combination of unknown coefficients is not possible in the non-linear regime, in contrast to the Kolosov-Muskhelishvili linear elastic solution. Thus, an analogous formulation of a HT-D finite element in the non-linear regime cannot be performed. Other multiscale methods are thus explored so as to formulate a non-linear perforated plate element.

Approach and main results obtained

The aim is to evaluate the cost-efficiency of several multi-scale approaches in the modelling of perforated plates (assemblies) in full-scale structure computations. The idea is first to perform linear regime cost-efficiency comparisons and then non-linear evaluations. The Transformation Field Analysis (TFA) [2] approach is firstly considered for its ability to localize fields. This approach is frequently used for weak discontinuities at the material scale, but less often for strong discontinuities at the structure scale. A perforated plate can be considered as a heterogeneous material containing a strong discontinuity - a hole, and thus represents a two-phase material with properties distinct from those of the solid plate [3]. Here, the TFA and combined Mori-Tanaka - TFA approaches are disccussed in particular.

The TFA method is established by decomposing a Representative Volume Element V (RVE) into a set of sub-volumes V_r. Within each sub-volume, the strain and stress fields are assumed to be uniform. The passage between global (macroscopic) fields and local (microscopic) fields is then characterized by localization relations eq. (1) and homogenization relations eq. (2) [2].

$$\boldsymbol{\varepsilon}_r = \mathbf{A}_r : \mathbf{E} + \sum_{s=1}^{N} \mathbf{D}_{rs} : \boldsymbol{\varepsilon}_s^{an} \quad \text{and} \quad \boldsymbol{\sigma}_r = \mathbf{B}_r : \boldsymbol{\Sigma} + \sum_{s=1}^{N} \mathbf{F}_{rs} : \mathbf{L}_s : \boldsymbol{\varepsilon}_s^{an} \tag{1}$$

$$\mathbf{E} = \sum_{r=1}^{N} c_r \boldsymbol{\varepsilon}_r \quad \text{and} \quad \boldsymbol{\Sigma} = \sum_{r=1}^{N} c_r \boldsymbol{\sigma}_r \tag{2}$$

The strain and stress fields in the sub-volume V_r are denoted by $\boldsymbol{\varepsilon}_r$ and $\boldsymbol{\sigma}_r$, respectively, while $\mathbf{E}$ and $\boldsymbol{\Sigma}$ represent the macroscopic strain and stress fields. The tensors $\mathbf{A}_r$ and $\mathbf{B}_r$ are the localization tensors for strain and stress, respectively. The tensor $\mathbf{D}_{rs}$ ($\mathbf{F}_{rs}$) represents the influence of strain (or stress) in a sub-volume V_r due to an applied strain on a sub-volume V_s. $\mathbf{L}_s$ denotes the elastic stiffness tensor. $\boldsymbol{\varepsilon}_s^{an}$ represents the anelastic strain (plastic, thermal, etc.). The scalar c_r is the volume fraction of the sub-volumes V_r $(c_r = V_r/V)$.

Thanks to the equations (eqs. (1) and (2)), the TFA method can be used in two manners - Sequential or Integrated approach (fig. 3). The **Sequential** approach consists of building a macroscopic behavior model based on the localization and homogenization relations. The objective is to determine an analytical expression $\left(\boldsymbol{\Sigma} = \mathbf{L}^{hom} : \mathbf{E}\right)$ linking the macroscopic strain to the macroscopic stress based on the knowledge of phenomena occurring at a lower scale. This expression can be implemented in a finite element code as a classical phenomenological law. However, implementing this approach in practice is very delicate, making its utilization quite rare. The **Integrated** approach features three steps : (1) Localization step → (2) Constitutive equations at the local scale → (3) Homogenization step. This approach does not necessitate the explicit formulation of macroscopic constitutive equations, unlike the sequential approach.

However, achieving accurate results typically requires a substantial number of sub-volumes, which significantly increases computational costs.

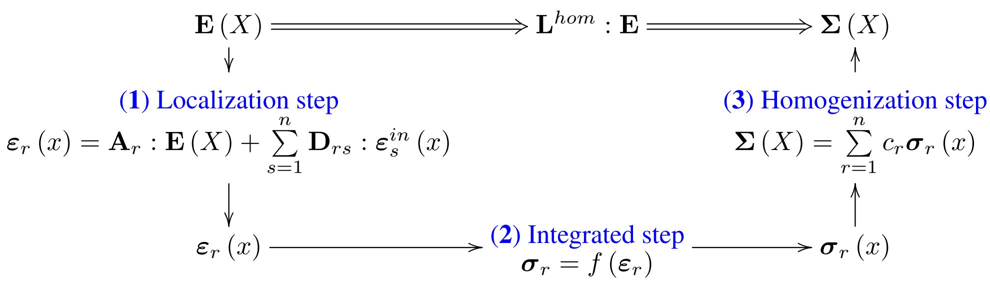

Figure 3: Sequential ($\Rightarrow$) and Integrated ($\rightarrow$) approach of TFA method

To model failure propagation in perforated plates, the integrated approach features more advantages than the sequential approach. Indeed, local fields are accessible to compute some criteria (such as yield stress or failure criteria), while the local fields are not accessible in the sequential approach. However, the cost associated with an integrated approach is very high. An alternative approach is to employ the Mori-Tanaka theory to eliminate the need for discretization of the RVE, thereby solving the problem at the structure scale (i.e., determining $\mathbf{\Sigma}$ from $\mathbf{E}$). Nonetheless, the local fields are not accessible when using Mori-Tanaka approach, contrary to the integrated TFA one. To overcome this limitation, a hybrid scheme combining the Mori-Tanaka method with 'fully' tensors calculated via the TFA approach is proposed.

The results of the first part of our work show that both these approaches provided accurate results in the homogenization of the linear perforated plate problem [4]. However, it appeared that the TFA approach was far costlier than the reference FEM solution. On the contrary, the combined Mori-Tanaka - TFA approach was several times faster than the reference. It is thus an interesting possibility in the cost-efficient treatment of multiscale problems in which it is required to access local fields.

The next steps will involve evaluating the effectiveness of Homogenization - TFA approaches for addressing materially and geometrically non-linear problems in perforated structures. Concurrently, the model reduction techniques such as the Proper Orthogonal Decomposition (POD) and neural networks will be considered to replace the Kolosov-Muskhelishvili solution within the hybrid-Trefftz framework.

List of communications

- Congrès des Jeunes Chercheurs en Mécanique (MECA-J), august 28-30, 2024, Online, France (Oral presentation)
- Congrès Français de Mécanique (CFM), august 25-29, 2025, Metz, France (Planned)
- Computational Modeling of Complex Materials Across the scales (CMCS), may 13-16, 2025, Champs-sur-Marne, France (Planned)

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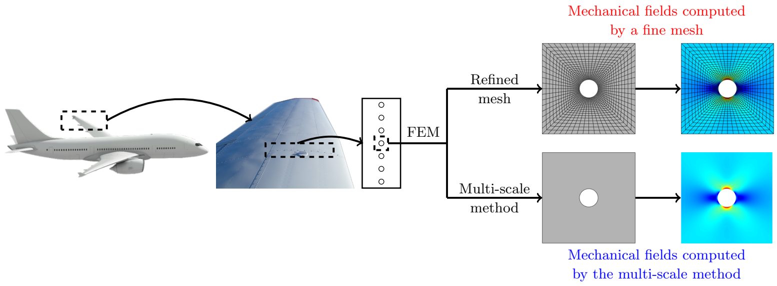

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